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A Novel Algorithm for Fast Sparse Image Reconstruction

G.UmaSameera*, M.V.R.Vittal**

**Department of ECE, G.pulla reddy engineering college, Kurnool,AP,India*
***Member IEEE, Department of ECE, G.pulla reddy engineering college, Kurnool,AP,India*

ABSTRACT: *A new technique for signal recovery and sampling is compressed sensing. It states that a sparse signal has relatively small number of linear measurements and it contains most of its information and these highly incomplete observations can exactly reconstruct that signal. The major challenge in practical applications of compressed sensing consists in providing efficient, stable and fast recovery algorithms which, in a few seconds evaluate a good approximation of compressible image from highly incomplete and noisy samples. In this paper compressed sensing image recovery problem using adaptive nonlinear filtering strategies in an iterative frame work and the resulting two-step iterative scheme convergence is proved. The several numerical experiments conform that he corresponding algorithm possesses the required properties of efficiency, stability and low computational cost and those of the state of the art algorithms are competitive to its performance.*

Index terms: *compressed sensing, sparse image recovery, nonlinear filters, median filters, L-minimization, total variation*

I. INTRODUCTION

In most image reconstruction problems, the images are not directly observable. Instead, one observes a transformed version of the image, possibly corrupted by noise. In the general case, the estimation of the image can be regarded as a simultaneous de-convolution and de-noising problem. Intuitively, a better reconstruction can be obtained by incorporating knowledge of the image into the reconstruction algorithm. The flows of data (signals and images) around us are growing rapidly today. However, the number of salient features hidden in massive data is usually much smaller than their sizes.

Hence data are compressible. In data processing, the traditional practice is to measure (sense) data in full length and then compress the resulting measurements before storage or transmission. In such a scheme, recovery of data is generally straight forward. This traditional data-acquisition process can be described as “full sensing plus compressing”. Compressive sensing (CS), also known as compressed sensing or compressive sampling, represents a paradigm shift in which the number of measurements is reduced during acquisition so that no additional compression is necessary. The price to pay is that more sophisticated recovery procedures become necessary. Compressed sensing is a new technique for recovery of the signal and sampling. It follows that signals that have a sparse representation in a transform domain can be exactly recovered from these measurements by solving an optimization problem which is represented as

$$\text{Minimize } \|a\|_1, \quad \text{subject to} \quad PW^T a = Pn \tag{1}$$

$a = Wu$ is coefficients vector of reconstruction u in that domain. Here P is an MXN matrix. Here M is very less than N. this matrix is necessary to possess restricted isometric property, n belongs to IR^N is the unknown signal, that is W belongs to $IR^{N\#N}$ orthogonal matrix of k-sparse transform domain. To obtain perfect recovery the number M of given measurements depends upon the N, k, P. K is orthogonal signal, P is acquisition matrix. If unknown signal n has sparse gradient, it can be recovered by problem (1) as

$$\min_u \| \nabla u \|_1 \quad \text{subject to} \quad PW^T a = Pn \tag{2}$$

Image recovery problem is suited for this formulation. Since many images can be modeled as piece-wise-smooth functions containing a substantial number of discontinuities. In real problems no need of exact measurements, so if the measurements are corrupted with random noise, namely we have

$$Y = Px + e \tag{3}$$

The original signal can be reconstructed with an error capable to the noise level by solving the minimum problem

$$\|a\|_1, \quad \text{subject to} \quad \|PW^T a - y\|_2^2 \le E^2 \tag{4}$$

$$\min_u \|d\,u\|_1, \quad \text{subject to} \quad \|Pu - y\|_2^2 \le E^2 \tag{5}$$

Sparse signals are an idealization that we rarely encounter in applications, but real signals are quite often compressible with respect to an orthogonal basis. This means that, if expressed in that basis, their coefficients exhibit exponential decay when stored by magnitude. As a consequence, compressible signals are well approximated by K –sparse signals and compressed sensing paradigm guarantees that from M linear measurements we can obtain a reconstruction with an error comparable to that of the best possible K –terms approximation within the scarifying basis

II. RECONSTRUCTION APPROACH

We set here our notation and state the results we will use in the following. Let S belongs to R(N1XN2) be a randomly generated binary mask, such that the point-to-point product with any v belongs to R(N1XN2) , denoted by S x v , represents a random selection of the elements of v, namely, we have

$$v_S = S.v \quad \text{With} \quad v_{S\,ij} = \begin{cases} v_{ij}, & if\ s_{ij} = 1 \\ 0, & if\ s_{ij} = 0 \end{cases} \tag{6}$$

Let T be an orthogonal transform acting on an image X We denote by

$$\mathrm{T}_S n = S \$ (\mathrm{T} n) \tag{7}$$

The randomly sub sampled orthogonal transform of.
Then the input data can be represented as

$$y = S \$ (\mathrm{T} n) = \mathrm{T}_S n \tag{8}$$

We want to find u belongs to R (N1xN2) that solves

$$\min_{u \in IR^{N1XN2}} F(u), \text{ subject to } \quad y = S \$ (\mathrm{T} n) = \mathrm{T}_S n \tag{9}$$

In the case of input data perturbed by additive white Gaussian noise with standard deviation

$$y = S \$ (\mathrm{T} n + e) = \mathrm{T}_S n + e_S \tag{10}$$

The problem can be cast as

$$\min_{u \in IR^{N1XN2}} F(u), \quad \text{subject to} \quad \|\mathrm{T}_S u - y\|_2^2 \le E^2 \tag{11}$$

$$E^2 = Z^2 (M + 2\sqrt{2M}) \tag{12}$$

To overcome this problem we use the well known penalization approach that considers a sequence of unconstrained minimization sub problems of the form

$$\min_{u \in IR^{N1XN2}} \{F(u) + \frac{1}{2Xk} \|\mathrm{T}_S u - y\|_2^2\} \tag{13}$$

The convergence of the penalization method to the solution of the original constrained problem has been established (under very mild conditions). Unfortunately, in general, using very small penalization parameter values makes the unconstrained sub problems very ill-conditioned and difficult to solve. In the present context, we do not have this limitation, since we will approach these problems implicitly, thus, avoiding the need to deal with ill-conditioned linear systems.
The corresponding bound constrained two-step iterative algorithm is the following:

$$\begin{cases} u_n = u_n + Y\mathrm{T}\,S^T(y - \mathrm{T}\,S u_n) \\ u_{n+1} = \operatorname{argmin}_{u\,!\,C}\{F(u) + \frac{1}{2X}\|u - v_n\|_2^2\}. \end{cases} \quad (14)$$

III. RECONSTRUCTION ALGORITHM

The proposed penalized splitting approach corresponds to an algorithm whose structure is characterized by two-level iteration. The general scheme of the bound constrained algorithm is following.

Algorithm NFCS-2D:

Step A-0: initialization
Given F . , y, $\mathrm{T}\,S$, $Y > 0$, $Z > 0$, $0 < r < 1$, $Toll \ \$\ 0$, $X_{\min}$ and X_0 such that $0 < X_{\min} \ \#\ X_0$.
Set $k = 0$, $u_{0,0} = 0$ and $X_{0,0} = X_0$.
Step A-1: start with the outer iteration

While $(X_{k,0} > X_{\min}$ *and* $\|\mathrm{T}\,S u_{k,0} - y\|_2 > Toll)$
Step B-0: Start with the inner iterations
$i = 0$;
Step B-1:
Updating Step:
$v_{k,i} = u_{k,i} + Y\mathrm{T}\,S^T(y - \mathrm{T}\,S u_{k,i})$
Constrained nonlinear filtering step:
$u_{k,i+1} = \operatorname{argmin}_{u\,!\,C}\ \{1/2X_{k,i}Y\|u - v_{k,i}\|_2^2 + F(u)\}$
Convergence test:
$if\ |F(u_{k,i+1}) - F(u_{k,i})|/F(u_{k,i+1}) \ \$\ ZX_{k,i}$
$i = i + 1$
$m_{k,i} = m_{k,i-1}$ go to step B-1
Otherwise go to step A-2.
Step A-2: Outer Iteration Updating
$k = k + 1$
$X_{k,0} = r.X_{k-i,i}$
$u_{k,0} = u_{k-1,i+1}$
endwhile
Terminate with $u^{k,0}$ as an approximation of x

Remark: The automatic stopping criterion of the outer loop depends upon which problem we are considering. If we want to recover an exactly sparse gradient image from noisy-free acquisitions the parameter Toll can be set to 0, and with X (min) of the order of the machine precision, we should obtain a numerically exact reconstruction. On the other hand, if we deal with compressible images or noisy data the stopping rule is governed by Toll.

Inducing norm F (.) can be chosen according to the characteristics of the reconstruction problem. In several nonlinear 1-D filters have been widely experimented for different compressed sensing signal reconstruction problems, and their capabilities and efficiency have been analyzed. In this context, we are mainly concerned with the image reconstruction problem and, since many real images can be well approximated with sparse gradient signals, we have only considered the choice, namely the case in which F (.) represents the total variation of the image.

IV. NUMERICAL EXPERIMENTS

Several numerical experiments reported as the effectiveness of the proposed image reconstruction algorithm that highlights its reconstruction capabilities, stability and speed. This choice is motivated by the need to give an objective quantitative evaluation of the effectiveness of the proposed algorithm by using reconstructed image quality.

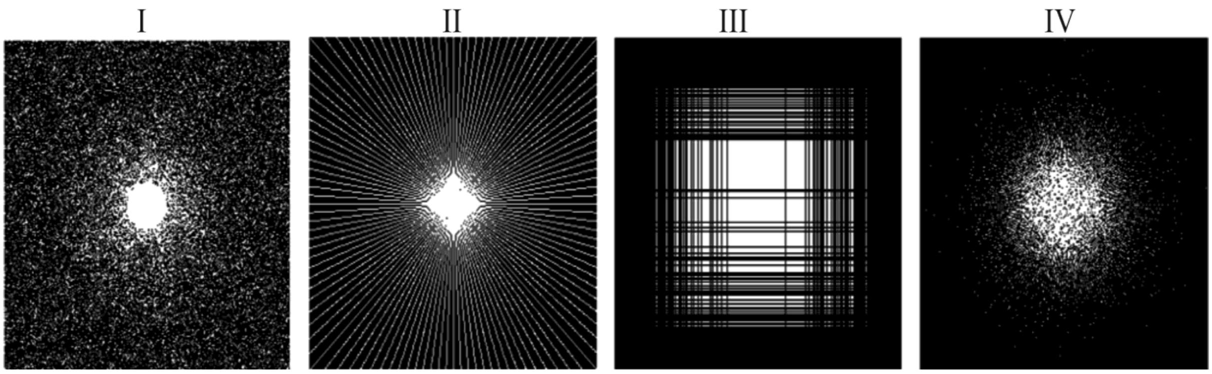

Fig: 1. 256# 256 Acquisition masks.

I: Sparse MRI masks corresponding to 75% under sampling. II: Radial mask with 60 rays corresponding to 77% under sampling.

III: 2-D tensor product Gaussian masks c 77% under sampling. IV: 2-D Gaussian masks corresponding to 90% under sampling.

Reconstructed images visual inspection is not really enough to compare the performance of different reconstruction algorithms.

The PSNR value is used to evaluate the quality of image

$$\mathrm{PSNR} = 20\log_{10}\frac{R}{\mathrm{RMSE}}$$

with

$$\mathrm{RMSE} = \frac{\mathrm{Error}}{\sqrt{N_1 \times N_2}}$$

Where R > 0 is the maximum value of the image gray level range and

$$\mathrm{Error} = \|u - x\|_2 = \sqrt{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(u_{i,j} - x_{i,j})^2}.$$

By using eight neighbour pixels we have used both unisotropic and isotropic descrete approximations of the total variation depicted in fig.1. the PSNR values that we give in different experiments refet to the first minimum energy reconstruction.

Since we have experimentally seen that it is not important to find a very accurate solution of the variation problem, for all the experiments we have fixed to four the number of iterations of the isotropic estimate yielded by the digital total variation filter. This choice represents a good compromise between accuracy and efficiency.

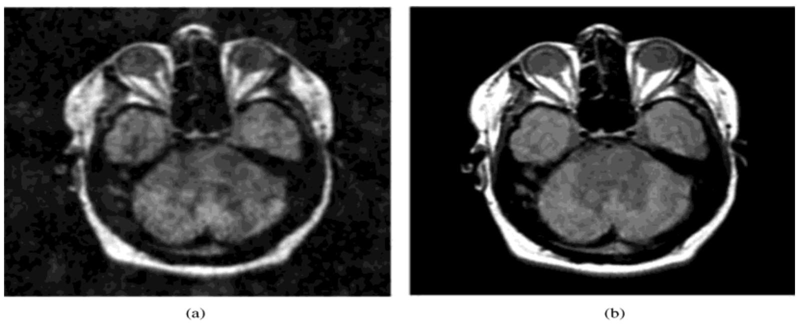

Fig: 2 Head image reconstruction from exact data, iterations

(a) Minimum energy reconstruction

(b) Isotropic reconstruction after 123 iterations

All the experiments are performed using sub sampled frequency acquisitions, but, in order to demonstrate the capabilities of our nonlinear filtering method, we have tested it using four different acquisition strategies corresponding to the masks given in Fig. 1. More precisely, mask I is evaluated using the free software Sparse, mask II is a classic 60 ray mask, mask III is obtained as a tensor product of two 1-D-Gaussian masks ,and mask IV is generated as 2-D normally distributed random points. The term nonlinear approximation error relative to the Haar basis and evaluated using the reconstructions obtained using the isotropic TV estimates are shown in Fig.2. In the last series of experiments we applied our nonlinear filtering strategy to recover the 256x256 Head image.

V. RECONSRRUCTION OF AN IMAGE RESULTS

Reconstruction of head image using nonlinear filtering is more efficient. Here we are using a head image and applying the randomly generated binary mask to that image, then the noisy image represented which is shown in the fig.3. By using NFCS-2D we will get the reconstructed image which is same as the original image. Here we are using some parameters in proposed algorithm. The first parameter is the starting value of the penalization parameter X. the reducing rate of X is r. the value of r is belonging to the interval [0.25- 0.8]. in practical, we have used a small value r = 0.25 for noise free sparse gradient case, and r = 0.4 or r = 0.8 for other cases. The second parameter is toll, used to stop the algorithm both for compressible images and clean data and for all cases of noisy data. When the data is noisy the noise level E is known, the possible choice of toll could be toll = E. since the value of E often overestimates the error norm, we have used a more flexible stopping criterion, setting toll= f.E, with f= [0.4, 1]. The choice of f=1 would stop the algorithm too early without exploiting its de-noising capabilities. The thord parameter is Z, which represents a mean for tuning the precision request in the inner iterations. A higher precision is responsible for an increase in the computing time, but can produce a more accurate reconstruction. So, in the attempt to find a good compromise between speed and reconstruction quality, we have used values of Z for sparse gradient images and exact data smaller than for compressible images and noisy data, that is Z=0.05 or Z= 1,respectively. Regarding the choice of the parameter Y we have always set Y= 1, even if a suitable greater value could be used to speed up the convergence of the algorithm.

Fig3: reconstruction of image from mask image

I. Noisy image, here randomly generated binary mask is used for original image,

II. Head image reconstruction from noisy image using NFCS-2D

The results of the reconstructed sparse image practically in NFCS, FRICS and NFCS-2D represented by applying the X=0.5 and Y=0.95 the psnr values become as

Elapsed time is 2.019534 seconds.

Elapsed time is 2.124597 seconds.

Elapsed time is 1.061977 seconds.

PSNR1 = 151.0998, PSNR2 = 218.9815, PSNR3 = 73.0840

Apply the values of X=0.1 and Y=0.50 then the results are

Elapsed time is 1.938029 seconds.

Elapsed time is 1.786954 seconds.

Elapsed time is 0.982569 seconds.

PSNR1 = 76.2816, PSNR2 = 129.7083, PSNR3 = 53.4428. in the three nonlinear filtering algorithms the NFCS-2D is more efficient.

VI. CONCLUSION

We have proposed an efficient iterative algorithm for the solution of the compressed sensing reconstruction problem, based upon a penalized splitting approach and an adaptive nonlinear filtering strategy and its convergence property has been established. We remark that, even if we have analyzed the sparse gradient case with under sampled frequency acquisitions, our approach is completely general, and works for different kinds of measurements and different choices of the function. The capabilities, in terms of accuracy, stability, and speed of NFCS-2D, are illustrated by the results of several numerical experiments and comparisons with a state of the art algorithm. In fact, since this function plays the role of the penalty function in the variation approach of the image de-noising problem, It is possible to exploit the different proposals of the de-noising literature in order to select new filtering strategies, perhaps more suited to the different practical recovery problems.

Examples of the use of other filtering strategies, even if considered in a different context. A lot of work remains to be done. In particular, a much more detailed theoretical study is necessary to find an objective automated way of selecting good values for the free parameters of the algorithm. At present, as far as we know, no similar analysis has been performed, even for the best state of the art algorithms.

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